Lambert meets Limon: An LDP at a CBL with a Quiz at the End

by
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Among the items on the NGS datum revision agenda for 2022 is an offer to develop Low Distortion Projections (LDPs) as requested by individual states. Covering relatively small areas (NGS has formulated rules for this), they are likely to be tangent projections instead of the secant projections used for both NAD 83 and NAD 27 state planes.

One way to explore tangent LDPs is to develop one and compare some ground measurements to grid distances. Since Calibration Base Lines have published measured distances and sometimes have NGS data sheets for their marks, they are a convenient resource. To be a good candidate, the CBL must also have been measured recently and it must be at an elevation that makes state plane grid distances significantly different from ground distances.

The Limon, Colorado CBL meets those criteria. Its marks have NGS data sheets containing NAD 83(2011) coordinates and the ellipsoid height of CBL 0 is 1610.152 meters. It was re-measured in 2014.

The marks are CBL 0 (KJ0588), CBL 400 (KJ0593), and CBL 1400 (KJ0594). Also, a 150 mark is listed in the CBL data, but there is no data sheet for it.

The grid origin for the Limon CBL LDP is CBL 0. The LDP is developed in accordance with the Lambert presentation in NGS Manual 5, modified to accommodate a single standard parallel.

For the projection to be tangent to the ground at CBL 0, the combined factor (CF) at that point has to equal one. The combined factor is the product of the elevation factor (EF) and the scale factor (SF). The elevation factor on the data sheet for CBL 0 is 0.99974744, so the scale factor can be computed:

$$CF = EF * SF$$

$$1 = 0.99974744 * SF$$

$$SF = 1/0.99974744 = 1.00025262380$$

Rounding the scale factor to 1.0002526 makes the distortion at the grid origin −0.020 parts per million with no effect on the Northing and Eastings of the points, so the practical effect is nil. However, the additional digits were helpful in eliminating system bugs, so I kept them.

Table 1 compares the mapping constants for the Limon CBL LDP to those of the Colorado state plane Central Zone 0502. Some of the defining constants for the state plane projection are computed constants for the LDP and vice versa. The table is in the current NGS format for a Lambert state plane, with the exceptions marked by bold print and asterisks.

The Colorado projection is defined by the standard parallels, Bs and Bn; the *true projection origin*, Bb; the longitude of the central meridian, Lo; the Northing at the true projection origin, Nb; and the Easting for the central meridian, Eo. Values for these numbers were selected by NGS.

The Limon projection is defined by the latitude of its central parallel, Bo; the scale factor at the central parallel, ko; the longitude of the central meridian, Lo; the Northing for the *grid origin*, No; and the Easting for the central meridian, Eo. Values for these constants were selected by the author.

Table 1. Comparison of Constants for Limon CBL LDP and Colorado Zone 0502

Defining Constants		
	Limon CBL LDP	Colorado Central Zone 0502
Bs=	NA	38:27
Bn=	NA	39:45
Bb=	**39:02:35.78945***	37:50
Lo=	103:39:50.67931	105:30
Nb=	0.0000	304,800.6096
Eo=	100,000.0000	914,401.8289

Computed Constants		
	Limon CBL LDP	Colorado Central Zone 0502
Bo=	**39.2684051778****	39.1010150117
SinBo=	0.632954054800	0.630689555225
Rb=	7,838,789.7339	7,998,699.7391
Ro=	7,813,789.7340	7,857,977.9317
No=	**25,000.0000****	445,522.4170
K=	12,498,366.6842	12,518,269.8410
ko=	**1.00025262380****	0.999935909777
Mo=	6,362,619.2189	6,360,421.3434
ro=	6,375,457	6,373,316

***Computed constant. **Defining constant. NA means Not Applicable.**

The Limon CBL LDP could have been designed by using Bb, the true grid origin, as a defining constant. Using a value such as 39:03 would have cleaned up the constants somewhat. However, doing that while preserving No = 25,000 would have required making Nb a computed constant, equal to 746.7963. Cleaning up one constant can dirty up another.

Thus, there are options and trade-offs inherent in the LDP design process. Taking one more step into the details, note that defining Bb as 39:02 while keeping everything else the same would make Nb negative.

LDP coordinates and more

Table 2 compares the LDP coordinates of the Limon CBL marks to the Colorado state plane coordinates for those marks. Note that, in the LDP, each point has a unique combined factor just as it does in the state plane. This is one of the ways that an LDP differs from the application of a project-wide combined factor. Grid distances computed from LDP coordinates will be very close to ground distances, but they can be made even closer by applying state plane distance computation procedures to them.

Table 2. Comparison of Coordinates Limon CBL LDP and Zone 0502

CBL 0	Limon CBL LDP	CO State Plane
Latitude	39° 16' 06.25864"	39° 16' 06.25864"
Longitude	103° 39' 50.67931"	103° 39' 50.67931"
Northing	25,000.000	465,705.526
Easting	100,000.000	1,072,818.305
Scale Factor	1.0002526238	0.99994016
Elev. Fact.	0.99974744	0.99974744
Comb. Factor	1.00000000	0.99968762
Distortion, ppm	0.000	-312.380
Convergence	0° 00' 0.0"	1° 09' 28.4"

CBL 400	Limon CBL LDP	CO State Plane
Latitude	39° 16' 19.01815"	39° 16' 19.01815"
Longitude	103° 39' 53.69975"	103° 39' 53.69975"
Northing	25,393.591	466,097.450
Easting	99,927.580	1,072,737.972
Scale Factor	1.00025263	0.99994035
Elev. Fact.	0.99974717	0.99974717
Comb. Factor	0.99999973	0.99968754
Distortion, ppm	-0.270	-312.460
Convergence	− 0° 00' 1.9"	1° 09' 26.5"

CBL 1400	Limon CBL LDP	CO State Plane
Latitude	39° 16' 50.90323"	39° 16' 50.90323"
Longitude	103° 40' 01.24918"	103° 40' 01.24918"
Northing	26,377.150	467,076.846
Easting	99,746.603	1,072,537.218
Scale Factor	1.00025265	0.99994082
Elev. Fact.	0.99974618	0.99974618
Comb. Factor	0.99999876	0.99968702
Distortion, ppm	-1.240	-312.980
Convergence	− 0° 00' 6.7"	1° 09' 21.8"

Other noteworthy items in Table 2 are the differences in magnitude of the LDP and state plane coordinates, the distortion values, and the convergence angles for the two projections. The LDP is designed for a very small area, so large-magnitude coordinates are not necessary. However, the state plane convention of having Eastings much larger than Northings is maintained. The LDP distortion values indicate that grid distances will be very close to ground distances. Convergence is a function of the difference between a point's longitude and the central meridian for some given latitude, so the LDP convergence angles are guaranteed to be nearer to zero.

Verifying distances and azimuths

Table 3 shows distances and azimuths for the points on the Limon CBL. Published distances can be verified at https://www.ngs.noaa.gov/CBLINES/BASELINES/co .

Table 3. Distances and Azimuths

Limon CBL

Distances in Meters

Coordinates			Grid Distance, Limon CBL LDP		Published Distance	
Point	Northing	Easting	From CBL 0	From CBL 400	From CBL 0	From CBL 400
CBL 0	25,000.000	100,000.000	--	--	--	--
CBL 400	25,393.591	99,927.580	400.1981	--	400.1971	--
CBL 1400	26,377.150	99,746.603	1,400.2686	1,000.0705	1,400.2719	1,000.0747

Azimuths					Arc-to-Chord (T – t)	
From	To	Grid Azimuth	Geodetic Azimuth	Decimal Grid Az	f3	T – t (Seconds)
CBL 0	CBL 400	349° 34' 27.4664"	349° 34' 27.5976"	349.5742962	39.26958661	-6.697E-09
CBL 0	CBL 1400	349° 34' 26.8662"	349° 34' 26.8660"	349.5741351	39.27253894	-8.200E-08
CBL 400	CBL 1400	349° 34' 26.5725"	349° 34' 24.6614"	349.5740707	39.27490181	-9.204E-08

Grid distances are within millimeters of the published ground distances. The largest difference, 4.2 mm, is from CBL 400 to CBL 1400. That's accuracy of about 1 : 238,000.

Examining the azimuths is instructive. Geodetic azimuths change over the course of the line, illustrating that geodetic lines do not have constant azimuths. Grid azimuths, adjusted for convergence, are close to geodetic azimuths.

A shortcut to an LDP

Software that can handle .prj files makes computing LDP coordinates easy, but it doesn't duplicate rigorous development of an LDP. While the software generates both forward and inverse coordinates, it may not display scale and elevation factors for the points nor the computed constants associated with the projection.

Nevertheless, here is a .prj for the Limon CBL LDP that works in DNRGPS and, perhaps, some other GIS and surveying systems. The defining constants from Table 1 are easily recognizable.

```
PROJCS["Lambert_Conformal_Conic",
GEOGCS["GCS_GRS 1980(IUGG, 1980)",
DATUM["D_unknown",
SPHEROID["GRS80",6378137,298.257222101]],
PRIMEM["Greenwich",0],
UNIT["Degree",0.017453292519943295]],
PROJECTION["Lambert_Conformal_Conic"],
PARAMETER["latitude_of_origin",39.268405177778],
PARAMETER["central_meridian",-103.66407758611],
PARAMETER["scale_factor",1.0002526238],
PARAMETER["false_easting",100000],
PARAMETER["false_northing",25000],
UNIT["Meter",1],
PARAMETER["standard_parallel_1",39.268405177778]]
```

Takeaways

My major takeaway from this project was the importance of the interplay between defining constants and computed constants for Lambert projections. The classifications and the values of the defining constants are design decisions that need careful consideration.

For 2022 secant Lambert state planes, NGS is changing the scale factor from a calculated constant to a defining constant. As a result, the two standard parallels become calculated constants. Knowing their values is neither necessary nor particularly useful and they won't be nice round numbers, so they won't be published.

The Northing at the true projection origin might be viewed in the same light. Changing it from a defining constant to a calculated constant would allow the choice of a nice round number for the Northing at the grid origin.

Exercises

Having read this far, perhaps you would consider doing a little work. The first exercise is easy; the last two are for folks who want to learn a bit more. The reward for correct answers is personal satisfaction. There is no penalty for incorrect ones, but shame on you if you don't at least do the first one.

1. Determine both the LDP and the geographic coordinates of CBL 150.

2. The Limon CBL is located at the Limon Municipal Airport (KLIC). The geodetic coordinates for the runway ends are 39-16-52.0738N, 103-40-02.5391W and 39-16-06.3973N, 103-39-51.7492W. The slope length of the runway is 4700 feet, rounded or truncated to feet, whichever the FAA requires. What are the Limon CBL LDP coordinates of the runway ends? What is the grid distance from runway end to runway end? Why is the geodetic distance more than a foot different from the grid distance? (Hint: Use the .prj file.)

3. The Limon CBL LDP optimizes grid versus ground near the Limon Municipal Airport. To judge its performance at some distance from its grid origin, determine Limon CBL LDP coordinates for KJ0231 and KJ0524 and find the LDP grid distance between them. Then use Colorado state plane coordinates and combined factors for the points to compute the ground distance between the two points. (Is the simple mean of the scale factors adequate?) Is the Limon CBL LDP sufficiently accurate for this computation?

4. Modify the .prj file given in the text to create an LDP that is tangent to the ground at KJ0524. What is the scale factor for the new LDP? the plane coordinates of Limon CBL 0?

More resources:

DNRGPS. A software package originally developed by Minnesota Department of Natural Resources, but now in the public domain. Designed to collect data from Garmin GPS receivers, it works with many other brands, including some survey grade receivers. It can transform coordinates using its extensive library of projection data and it accepts custom projections. Sometimes a bit temperamental, it is nonetheless a great little system. https://www.dnr.state.mn.us/mis/gis/DNRGPS/DNRGPS.html

ESRI Support. Here's a bit more about .prj files. https://support.esri.com/en/technical-article/000001897

Krakiwsky, E. J. Conformal Map Projections in Geodesy. http://www2.unb.ca/gge/Pubs/LN37.pdf

Explains the mysterious Qs, Qn, and Qo in Stem's publication and a host of other stuff.

Stem, James E. State Plane Coordinate System of 1983..
https://www.ngs.noaa.gov/library/pdfs/NOAA_Manual_NOS_NGS_0005.pdf

Note: Two necessary calculated constants for a Lambert projection are K and Ro, the mapping radii at the equator and the central meridian, respectively. Modifying secant state plane projections to single parallel projections can be accomplished by rewriting two equations on page 28 of this publication.

Since k0 is a defining constant for the single parallel projection, the equation that calculates its value (the second equation on the page) can be solved for Ro:

Ro = (k0*a)/(w0*tan(phi0))

Then solve the first equation on the page for K:

K = Ro*exp(Qo*sin(phi0))

With these constants calculated, the forward and inverse formulas can be used as written. These values are determined by GIS software from the information in the .prj file, but they can be calculated in spreadsheet implementations

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